Explore the potential of exponential growth

Mathematics could help you get a good deal at the bank, find true love and more. Paul Davis explains how.

The 'Exponential Growth' formula wasn't a gimmick after all!
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Few people love mathematics. A common refrain among students is, “Why do I have to learn this stuff? When will I need it?” But having a working knowledge of the basic concepts is essential in daily life as an adult. We use them when counting cash, calculating mortgage payments and filling out tax returns.

In fact, it was financial matters such as loans, interest payments and gambling that spurred the development of a lot of early mathematics. Negative numbers, for example, were needed to represent debt, and the mathematical rules for their use were worked out in India and the Islamic world by the 7th century.

One money problem that was carefully analysed in the 17th century concerned compound interest – a familiar enough concept today. Just like modern banks, the money lenders of the day competed for customers using interest rates as incentives. But when making comparisons the customer always has to be careful of the small print. Interest rates are normally expressed on an annual basis. For example, a simple 5% annual interest means that $100 investment becomes $105 at the end of one year. But if interest is credited, say, every six months, the customer gets a higher overall annual return.

To keep the arithmetic simple, imagine a bank that paid 100% annual interest (that would be nice!). If credited annually, that rate of interest would turn $100 into $200 at the end of the year. But if credited every six months, then $50 gets credited to the account after six months, so at the end of the year the original capital has earned $100, but the $50 credited after six months will itself earn $25 interest over the second half of the year. So by offering biannual compound interest, the bank would pay the customer $125 interest at the end of one year rather than $100. A customer who started with $100 would now have $225 in the account.

If the interest is paid quarterly, the deal is even better, amounting to a little over $244 at the end of the year. The more often the interest is credited, the higher the final total. But it is a process of diminishing returns: the total goes up by a smaller and smaller amount the more frequently you credit the interest. Crediting every day would yield a bit over $271. That is to say, the original capital will have been boosted 2.71 times.

All of which raises the question: what would be the upper limit to this compounding process? Mathematicians were pondering this even back in the 17th century. In 1683, the mathematician Jacob Bernoulli found the answer: 2.7182818... (the ellipsis indicates that this number is an unending decimal). It is an ***irrational number <https://cosmosmagazine.com/mathematics/the-square-root-of-2>*** and, like π, proved to be a fundamental mathematical constant that turns up in fields as diverse as accounting, physics, engineering, statistics and probability theory. It is such an important number it is given a letter all its own: *e*.

Peruse any textbook on science, engineering or economics, and you will see the symbol *e* scattered throughout. It is most often used in connection with “exponential growth” – a term that has entered the popular lexicon, though it is often misused. The correct meaning refers to a specific type of rapid, runaway growth in which a quantity doubles in a fixed time, and then doubles again, and again, *ad infinitum*. The population of bacteria in a dish, for example, will increase exponentially if their growth is unrestrained.

One familiar example of exponential growth is Moore’s Law, named after Gordon Moore, co-founder of Intel. After noticing in 1965 that the size of transistors was rapidly shrinking, which meant more of them could fit onto a computer chip, he predicted that processing power would double roughly every two years (and the price would drop by half). Remarkably, this exponential growth has remained more or less consistent for several decades, though nobody expects it to go on forever.

And *e* makes a surprise appearance in less obvious places, too. My favourite example is *e*’s application to the secretary problem. Imagine there are 100 applicants to be randomly interviewed for a secretarial job. At the end of each interview, the interviewer must give the applicant an irrevocable decision as to whether they’ve got the job. It would be risky to see them all, dismissing the first 99, because the 100th interviewee would have to be given the job regardless of quality.

The conundrum is this: to maximise the probability of getting the best candidate, how many should be interviewed before selecting the first remaining candidate who trumps the ones already seen? It turns out the answer is 100/*e*, or about 37. This result is worth remembering by people who like to play the dating game methodically.

So mathematical knowledge isn’t just useful at tax time. Perhaps if more people knew maths could help them find love, more would be willing to embrace it.

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***PAUL DAVIES** is a theoretical physicist, cosmologist, astrobiologist and best-selling author.*